NUMERICAL SIMULATION OF A SOLAR FLAT PLATE COLLECTOR : A MULTIDIMENSIONAL APPROACH

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CERTIFICATE

It is certified that the work contained in the thesis entitled "*Numerical Simulation Of A Solar Flat Plate Collector : A Multidimensional Approach*", by Mr. *Mahendra Prasad Agrawal*, has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

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Dedicated to

My Loving Parents

Abstract

This thesis presents a numerical simulation taking into account multidimensional modelization of the different elements of a flat plate collector, viz., cover, ducts, fluid flow, insulation etc. These different elements have been coupled in a global algorithm. The algorithm is presented, herein, in the form of a flow chart. In practice, the temperature of ducts,plate etc., is not uniform but varies from point to point. Finite difference method has been used to solve the governing differential equations, and the effect of the various operational parameters on the performance of the flat plate collector has been investigated.

Performance of a solar flat plate collector by the multidimensional and the conventional methods of analysis has been compared. The results indicate that the efficiency of a flat plate collector by the multidimensional approach is about 7 to 10 percent more in comparison to the conventional approach. It is found that parallel connection of the ducts is more efficient than the series connection. The experimental results obtained from literature agree closely with those obtained from the multidimensional analysis. Multidimensional analysis being more efficient can naturally lead to a reduction in the conventional flat plate collector area. It is also observed that there is a steeper rise in plate temperature in the direction of flow in the multidimensional approach than in the conventional approach. The results are presented in the form of graphs between various parameters.

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Nomenclature

A	area(m^2)
C	specific heat($J.kg^{-1}.K^{-1}$)
C_p	specific heat at constant pressure($J.kg^{-1}.K^{-1}$)
D_i	inner diameter of the duct(m)
D_o	outer diameter of the duct(m)
h	heat transfer coefficient($W.m^{-2}.K^{-1}$)
i,j,k	indicator of i,j and k_{th} spatial position
k	thermal conductivity($W.m^{-1}.K^{-1}$)
I_o	solar radiation flux on the collector per unit area($W.m^{-2}$)
$\dot{m}$	total air mass flow flux($kg.s^{-1}$)
n	number of nodal points
N	number of ducts
q	rate of heat transfer per unit of surface($W.m^{-2}$)
Q	rate of heat transfer(W)
S	amount of heat flux absorbed by the absorber plate($W.m^{-2}$)
T	temperature(K)
U	heat transfer coefficient($W.m^{-2}.K^{-1}$)
x,y,z	spatial coordinates

Greek Symbols

α	absorptivity of the absorber plate
β	collector tilt
γ	surface azimuth
δ	thickness(m)
ϵ	emissivity
ϕ	latitude
ρ	reflectivity
θ	angle of incidence of radiation on a tilted surface
θ_z	angle of incidence of radiation on a horizontal surface
τ	transmissivity of the glass cover
σ	Stefan-Boltzmann constant
η	efficiency

Subscript

a	ambient
b	bottom
c	cover
d	ducts
f	fluid
i	insulation
in	inlet conditions
l	loss
m	mean
o	output conditions
p	absorber plate
u	useful
w	wind

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Chapter 1

Introduction

1.1 Importance of solar energy

While conventional sources of energy cannot be neglected, it is imperative to develop programmes in the area of renewable sources as they not only help conserve scarce conventional sources of energy but contribute immensely to improvement of environment,emloyment generation,upgradation of health and hygiene,etc. The potential for energy generation from these sources - sun,wind,biomass,etc. - is immense.

Taking a global energy view, it is felt that solar energy is one of the most promising alternate sources because of its diffuse nature and world wide availability.Solar energy has a great future in a tropical country like India, where many places receive a peak solar flux of the order of 1 kW/m^2. The need for solar energy has become especially relevant in the wake of the energy crisis in the rural sector.In India there are still 100,000 villages where darkness is not dispelled by electricity.These villages,due to their remote location, are not likely to have access to electricity supply from the national grid for a very long time.In the rural areas,domestic energy consumption,especially for cooking, constitutes a significant part of energy

demand. The other uses of energy are for lighting,water heating,irrigation,post-harvesting operations and village industries like brick kilns,flour mills,rice mills and agro processing units.

Some of the viable uses of solar energy are as follows :

1. **Solar water heating**

   Many flat plate and concentrating collectors have been designed for their usefulness in water heating. They consist of a box with glass covers,covering a metallic absorber plate carrying water in the ducts. The water is heated by the solar energy absorbed by the blackened absorber plate.

2. **Solar cooking**

   A solar cooker can be easily designed and fabricated.The simplest type is a box with a glass cover facing the sun. Mirrors can be provided to give some degree of concentration,which would bring the food to be cooked, up to the cooking temperatures quickly.

3. **Space heating**

   Space heating is generally employed in cold countries where keeping the house warm is a big problem.This is normally achieved by using solar air-heaters(active solar-heating) or by designing the house itself to trap solar energy(passive solar-heating).

4. **Conversion to electricity**

   Electricity can be generated directly from sunlight with the help of solar photovoltaic (SPV) technology.The basis is the photoelectric effect.The atoms of certain metals, such as selenium, possess electrons that are easily knocked out of place by light energy. Another metal that is in contact with the electron-emitting metal collects the electrons and passes them along into

wires in a steady stream,while other electrons from the wires flow in to replace them.Thus,an electric current is established.

5. **Solar distillation**

   One of the major problems in many parts of the world is the non-availability of fresh water.Solar stills can convert saline or brackish water into fresh potable water easily,by the process of evaporation and condensation.

Some other uses of solar energy worth mentioning are - solar refrigeration and air conditioning,solar furnaces,solar pumps,heating sewage digestors in sewage treatment,solar pond,generation of mechanical power,drying grains or vegetables etc.

1.2 Solar collectors and their main components

In the last few decades,new solar collectors of different shapes have come up.The flat plate collectors have evoked considerable interest because they are the simplest among the solar energy collecting devices and have a wide range of potential for heating and cooling applications.They usually consist of the following parts :

1. **Absorber plate**

   It is a blackened metallic plate which is exposed to sun.It is made of a highly conducting material with a view to trap maximum amount of radiation incident upon it by transferring it to the fluid flowing through the ducts attached to the underneath of the absorber plate.

2. **Cover plate**

   It is a glass plate usually placed over the absorber plate to reduce heat losses.It allows the short wavelength radiation from the sun to pass through it and impinge on the absorber plate.Due to the heating up of the absorber

plate long wave radiations are produced.These radiations are prvented by the glass cover from passing into the atmosphere.Hence it leads to higher collection temperature for fixed heat losses.The cover plate may be one or two in number.

3. **Concentrator**

Now-a-days, most of the solar water heaters are accompanied by a concentrator which is used to increase the solar energy incident on the absorber plate.The purpose is to get higher collection temperatures. The concentrators are available in different shapes - parabolic, plane glass, cylindrical etc.

4. **Fluid**

It is a means to extract the heat absorbed by the absorber plate.This hot fluid may directly be used for heating purposes or may transfer its heat to another fluid through a heat exchanger.

5. **Insulation**

The bottom part of the absorber plate is separated from the ambient conditions by means of a layer of insulating material. This insulation prevents the loss of heat by the plate to the surroundings. Usually the insulation used are glass wool, wood wool, saw dust, thermocole etc.

Chapter 2

Literature Survey And The Present Work

2.1 Literature Survey

In most of the current bibliography the thermal behaviour of solar collectors is calculated on the basis of models that do not take into account the multidimensional and transient nature of the phenomena [1,2].When these aspects are analysed,the governing heat equations that characterise the thermal behavior of a solar collector are difficult to solve.Different models with simplified assumptions have appeared in an attempt to assess the transient behaviour of a collector.A review of these is presented by J.G.Smith [3].These models , however, do not take into account the multidimensional aspects of the problem.

The rapid and continuous advance of high speed digital computers makes it possible to develop more realistic physical models with the help of numerical techniques based on finite difference and finite elements.P.L.T.Brian [7] used a finite difference method of high-order accuracy for the solution of three dimensional transient heat conduction problems.L.H.Thomas [8] used an algorithn to solve elliptical problems

in linear difference equations over a network.Consequently some work dealing with numerical simulation of solar collectors was carried out.Saito et. al.[4] presented a numerical solution for a solar water heater.They considered a transient one dimensional model.Chiou [5] presented a bidimensional steady model to study the non-uniformity in the fluid flow distribution.Oliva et. al. [13] presented a numerical simulation of solar collectors in which they considered the effects of nonuniform and nonsteady boundary conditions.They also studied the influence of shadows due to external obstacles and cover supports of the collector.

2.2 Present Work

This thesis, however, presents a numerical simulation taking into account a multidimensional modelization of the different elements of a solar collector, viz., cover,duct,plate etc. and their coupling in a global algorithm. Finite difference technique for the solution of the governing partial differential equations has been employed.

The conventional one-dimensional analysis for a solar flat plate collector with water as the heating fluid is based on the following assumptions :

1. Performance is steady-state.
2. The headers cover a small area of the collector surface and can be neglected.
3. The headers provide uniform flow to the tubes.
4. There is no absorption of solar energy by the glass covers insofar as it affects the losses from the collector.
5. There is one-dimensional heat flow through the glass covers.
6. There is a negligible temperature drop through the glass covers.
7. There is one dimensional heat flow through the bottom insulation.

8. The sky can be considered as a black body.

9. Temperature gradient around the tubes can be neglected.

10. The temperature gradient in the direction of flow and between the tubes can be treated independently.

11. Properties are independent of temperature.

12. Dust and dirt on the collector are negligible.

13. Shading of the collector absorbing plate is negligible.

The present work, as mentioned above, takes into account the multi-dimensional aspect of a solar flat plate collector.Solar energy absorbed by the glass cover is also taken into account. Temperature variation in the absorber plate is assumed to be two dimensional, therefore there is no need to consider the independent variation of temperature in the direction of flow and between the tubes.Hence this multi-dimensional analysis, therefore, leads to the relaxation of five assumptions, viz., -4^{th}, 5^{th}, 6^{th}, 7^{th} and the 10^{th}.

Chapter 3

Problem Formulation

3.1 Solar Radiation

Total amount of hourly global radiation reaching the tilted surface at any instant[14] is given by

$$I_o = I_b R_b + I_d R_d + (I_b + I_d) R_r \tag{3.1}$$

where

$$\begin{aligned}
I_b &= \text{hourly beam radiation} \\
I_d &= \text{hourly diffuse radiation} \\
R_b &= \text{tilt factor for beam radiation} \\
&= \frac{cos\theta}{cos\theta_z} \\
R_d &= \text{tilt factor for diffuse radiation} \\
&= \frac{1+cos\beta}{2} \\
R_r &= \text{tilt factor for reflected radiation} \\
&= \rho(\frac{1+cos\beta}{2})
\end{aligned}$$

Total amount of heat flux absorbed by the absorber plate is given by

$$S = I_b R_b (\tau\alpha)_b + (I_d R_d + (I_b + I_d) R_r)(\tau\alpha)_d \tag{3.2}$$

where

$(\tau\alpha)_b$ = transmissivity-absorptivity product for beam radiation

$(\tau\alpha)_d$ = transmissivity-absorptivity product for diffuse radiation

3.2 A brief idea about the conventional method

Heat lost from a solar flat plate collector can be expressed in terms of an overall loss coefficient, U_l, defined by the equation

$$Q_l = U_l A_p (T_{pm} - T_a) \tag{3.3}$$

The heat lost from the collector is also the sum of the heat lost from the top and bottom of the collector. Thus

$$Q_l = Q_t + Q_b \tag{3.4}$$

where,

$$Q_t = \frac{\sigma A_p (T_{pm}^4 - T_{cm}^4)}{\frac{1}{\epsilon_p} + \frac{1}{\epsilon_c} - 1} + h_{p-c} A_p (T_{pm} - T_{cm}) \tag{3.5}$$

$$= \sigma A_p \epsilon_c (T_{cm}^4 - T_{sky}^4) + h_w A_p (T_{cm} - T_a) \tag{3.6}$$

$$Q_b = k_i A_p \frac{(T_{pm} - T_a)}{\delta_i} \tag{3.7}$$

The useful heat gained by the fluid is,

$$Q_u = \dot{m}C_p(T_{fo} - T_{fi}) \tag{3.8}$$

$$= F_R A_p[S - U_l(T_{fi} - T_a)] \tag{3.9}$$

where,

$$F_R = \frac{\dot{m}C_p}{U_l A_p}[1 - exp(-\frac{F' U_l A_p}{\dot{m}C_p})] \tag{3.10}$$

= a measure of thermal resistance encountered by the absorbed solar radiation in reaching the collector fluid.

= ratio of actual heat gain to the heat gain if the entire plate were at the fluid inlet temperature, T_{fi}

$$F' = \frac{1}{WU_l[\frac{1}{U_l(W-D_o)\phi+D_o} + \frac{1}{\pi D_i h_f}]} \tag{3.11}$$

$$\phi = \frac{tanh[m\frac{(W-D_o)}{2}]}{m\frac{(W-D_o)}{2}} \tag{3.12}$$

$$Q_l = SA_p - Q_u \tag{3.13}$$

The above equations can be used to solve the temperatures of plate ,cover and the fluid .Efficiency (η) is defined as

$$\eta = \frac{Q_u}{I_o A_p} \tag{3.14}$$

3.3 Governing Equations for the multidimensional approach

This section provides a more appropriate and accurate multidimensional approach for the analysis of a liquid flat plate solar collector. The mathematical description of

the multidimensional phenomena in the different collector zones is presented here. Properties of the fluid and the materials have been considered to be independent of temperature and invariant with time. Depending on the relative values of the different dimensions, thermal energy transfer through the solid zones has been considered to be two or three dimensional e.g. the heat exchange through the insulation has been considered to be three dimensional while that through the plate has been considered to be two dimensional. Temperature variation of the fluid flowing inside the ducts is taken as one dimensional in the flow direction. Finally, the free convection in the air gap zone,i.e., between the absorber plate and the flat plate collector cover is taken into account by means of an empirical expression.

1. **Cover**

The cover thickness is so small that it is reasonable to consider a uniform temperature through it.Hence, we get a two dimensional distribution of temperature T_c(x,y).The governing equation can be obtained from an energy balance in a small volume of thickness δ_c, as

$$k_c[\frac{\partial^2 T_c}{\partial x^2} + \frac{\partial^2 T_c}{\partial y^2}] + \frac{q_{p-c}}{\delta_c} + \frac{q_{a-c}}{\delta_c} + q_{abs} = 0 \qquad (3.15)$$

where,

$$q_{p-c} = h_{p-c}(T_{pm} - T_c) + \frac{\sigma(T_{pm}^4 - T_c^4)}{\frac{1}{\epsilon_p} + \frac{1}{\epsilon_c} - 1}$$

$$q_{a-c} = h_w(T_a - T_c) + \sigma\epsilon_c(T_{sky}^4 - T_c^4)$$

$$q_{abs} = I_o(1 - \tau_a)/\delta_c$$

Here q_{p-c} is the heat flux which is transmitted from the absorber plate to the cover.It consists of two components - the radiation and the convection. h_{p-c}, which depends on the temperature difference between the absorber and the cover plates, is calculated by using an empirical expression [12] because

a complete and a rigorous analysis of the recirculating flow in the air gap zone would produce an excessive penalization in the CPU time. q_{a-c} denotes the rate of convective and radiative heat transfer per unit of cover surface that arrives at the cover from the surroundings. q_{abs} is the amount of solar radiation heat transfer per unit volume absorbed by the cover. It has been calculated using Bouger's law for a partially transparent medium.

Adiabatic condition is considered on the boundary ,i.e.

$$\frac{\partial T_c}{\partial n} = 0 \tag{3.16}$$

2. **Insulation**

The temperature distribution in insulation is considered to be three dimensional.The governing heat equation is written as

$$k_i[\frac{\partial^2 T_i}{\partial x^2} + \frac{\partial^2 T_i}{\partial y^2} + \frac{\partial^2 T_i}{\partial z^2}] = 0 \tag{3.17}$$

There is no heat generation or absorption inside the insulation.The boundary conditions are :

- Faces exposed to the atmosphere

$$-k_i(\frac{\partial T_i}{\partial n}) = h_w(T_i - T_a) \tag{3.18}$$

- Face in contact with the absorber plate

$$T_i = T_p \tag{3.19}$$

3. **Fluid**

The fluid flow in the duct is considered to be one dimensional i.e. average values of temperature are used in each cross-sectional area of the fluid stream. The governing heat equation of the fluid flow is :

$$\frac{\dot{m}}{N} C_p \frac{dT_f}{dy} = q_w \tag{3.20}$$

where,

$$q_w = h_f \frac{\pi D_i}{2} [(T_p - T_f) + (T_d - T_f)] \tag{3.21}$$

Here q_w is the flux of heat transfer by convection from duct walls to the fluid. The inlet boundary condition for the duct is

$$For \quad y = 0 \qquad\qquad T_f = T_{fi} \tag{3.22}$$

4. **Duct**

The ducts are considered to be circular . The top portion of the duct is considered to be at the same temperature as that of the plate in contact with the duct at that point. This is considered to be reasonable as the thermal conductivity of both the duct and the plate materials are very high. For the bottom portion we consider the variation of temperature in the direction of flow as other dimensions are very small as compared to the length of the duct.The governing equation for a particular duct is :

$$k_d \frac{d^2 T_d}{dy^2} + \frac{q_{f-d}}{\delta_d} + \frac{q_{i-d}}{\delta_d} = 0 \tag{3.23}$$

where,

$$q_{f-d} = h_f (T_f - T_d)$$

$$q_{i-d} = h_w (T_a - T_i)$$

q_{f-d} is the amount of convective heat flux flowing from the fluid to the duct. q_{i-d} is the amount of heat flux flowing from the atmosphere to the duct.In actual case it is negative.Here q_{i-d} includes only that much of heat which is

transferred to a particular duct from the atmosphere. The rest of the heat is transferred to the plate and other ducts.

5. **Plate**

As in the case of cover,here also we consider a two dimensional variation in temperature i.e. $T_i(x, y)$. The governing heat equation can be obtained, as before, by a heat balance on an element of thickness δ_p, as

$$k_p[\frac{\partial^2 T_p}{\partial x^2} + \frac{\partial^2 T_p}{\partial y^2}] + \frac{q_{f-p}}{\delta_p} + \frac{q_{i-p}}{\delta_p} + \frac{q_{c-p}}{\delta_p} + \frac{S}{\delta_p} = 0 \tag{3.24}$$

where,

$$q_{f-p} = h_f(T_f - T_p)$$

$$q_{i-p} = h_w(T_a - T_i)$$

$$q_{c-p} = h_{p-c}(T_{cm} - T_p) + \frac{\sigma(T_{cm}^4 - T_p^4)}{\frac{1}{\epsilon_c} + \frac{1}{\epsilon_p} - 1}$$

Here q_{f-p} represents the amount of heat transfer per unit area transferred to the plate by the fluid flowing through the ducts. q_{i-p} represents the amount of heat flux lost to the atmosphere through the insulation.This is equivalent to the bottom heat loss from the absorber plate in case of the conventional one dimensional analysis. 'S' is the amount of solar radiation absorbed by the absorber plate.

Adiabatic condition is considered on the boundary ,i.e.

$$\frac{\partial T_p}{\partial n} = 0 \tag{3.25}$$

Chapter 4

Numerical Modelization

In this chapter, the methods and the discretized equations used in different collector zones are presented. These zones are the cover, insulation, fluid, the duct and the absorber plate.The global algorithm for the coupling solution is indicated by a flowchart (Fig.4.2). Implicit finite difference method is used for the solution of the various equations. In the method the entire region is divided into small grids of finite size. Temperatures are calculated on all the nodal points.Partial derivatives are expressed in the following forms:

$$\frac{\partial T}{\partial x} = \frac{T(i+1)-T(i-1)}{2\Delta x}$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{T(i+1)-2T(i)+T(i-1)}{(\Delta x)^2}$$

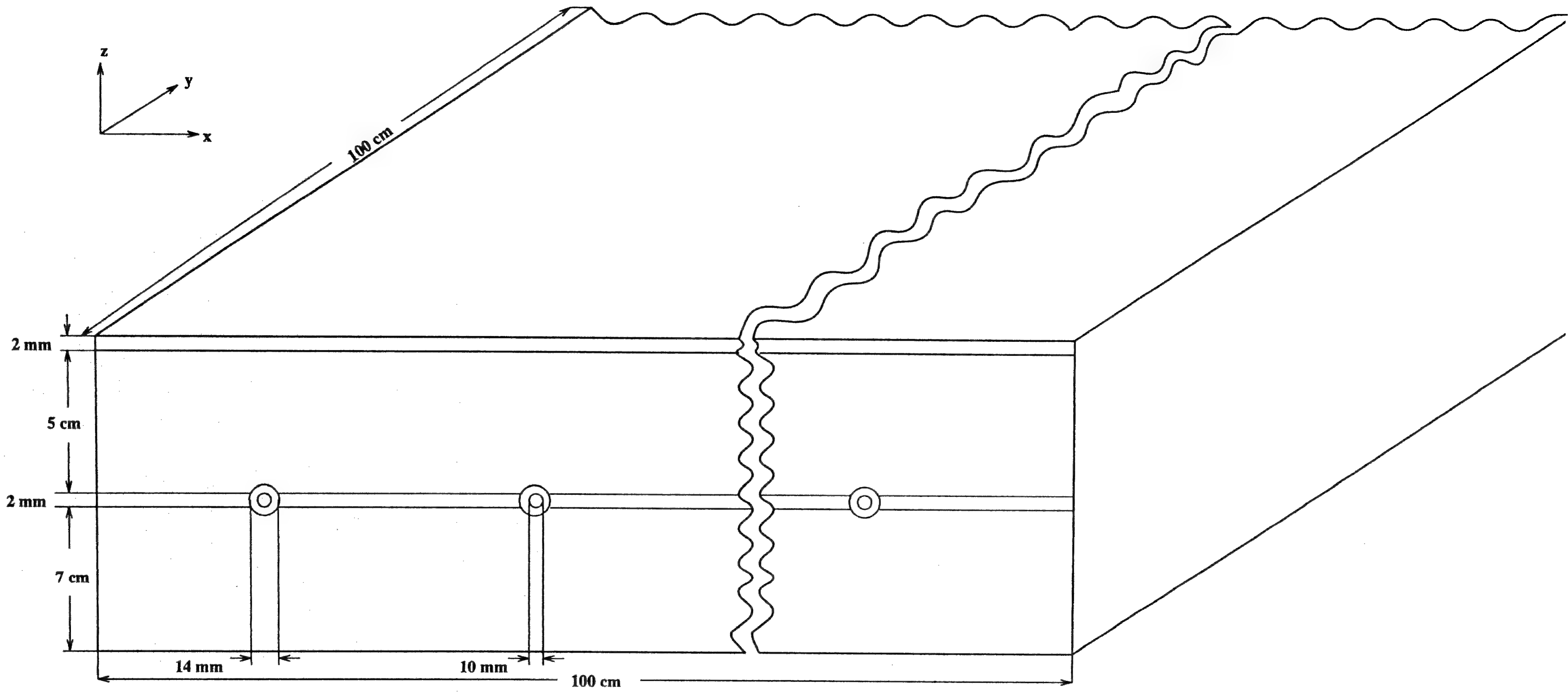

Figure 4.1: Flat plate collector configuration

1. **Cover**

The governing equation for transfer of heat in the cover is :

$$k_c[\frac{\partial^2 T_c}{\partial x^2} + \frac{\partial^2 T_c}{\partial y^2}] + \frac{h_{p-c}(T_{pm} - T_c)}{\delta_c} + \frac{\sigma(T_{pm}^4 - T_c^4)}{\delta_c(\frac{1}{\epsilon_p} + \frac{1}{\epsilon_c} - 1)}$$
$$+ \frac{h_w(T_a - T_c)}{\delta_c} + \frac{\sigma\epsilon_c(T_{sky}^4 - T_c^4)}{\delta_c} + q_{abs} = 0 \qquad (4.1)$$

Discretization is carried out by means of a regular two dimensional grid. Partial derivatives of temperature with respect to coordinates are expressed in terms of temperatures at the nodal points. For each one of the grid point we obtain a discretized equation. The boundary conditions can be written in the discretized form as:

$$\begin{array}{llcll} for & x & = & 0 & T_c(-1,j) = T_c(1,j) \\ for & y & = & 0 & T_c(i,-1) = T_c(i,1) \\ for & x & = & W & T_c(n_x+1,j) = T_c(n_x-1,j) \\ for & y & = & L & T_c(i,n_y+1) = T_c(i,n_y-1) \end{array}$$

The equations for the discrete points can be written in the form of a matrix.

$$[A]_{(n_x*n_y)*(n_x*n_y}[T_c]_{(n_x*n_y)*1} = [C]_{(n_x*n_y)*1} + [D]_{(1*1)}[T_c]^4_{(n_x*n_y)*1}$$

where A,C and D are matrices having elements which are independent of the temperature of the cover . We assume an initial temperature distribution of the cover plate,which is substituted on the right hand side of the above equation.Then by using Gauss elimination method, we obtain a new temperature distribution in the cover plate. This new distribution is then placed on the right side of the above equation. Thus by iteration, we get the exact temperature profile.

2. **Insulation**

The governing heat equation for the insulating material is (3.15).This equation can be discretized by expressing the partial derivatives in terms of temperatures at the nodes.Hence we get a set of equations for each nodal point.The boundary conditions can also be discretized.

$$For \qquad z = \delta_i \qquad\qquad T_i(i,j,k) = T_p(i,j)$$

For all the other faces there is heat transfer by convection to the atmosphere.The corresponding boundary condition can be discretized by using the difference formula for first order derivatives.e.g.

$$For \qquad x = 0 \qquad - k_i[\frac{T_i(1,j,k) - T_i(-1,j,k)}{2(\Delta x)}] = h_w(T_i - T_a)$$

$$T_c(-1,j,k) = T_c(1,j,k) + \frac{2(\Delta x)h_w(T_i - T_a)}{k_i}$$

This expression for $T_i(-i,j,k)$ is substituted in the discretized equation to obtain the equation for the boundary points at x = 0.Similar discretization method is followed for other faces in contact with the atmosphere.

The equations for the node points can be expressed in the following matrix form :

$$[A]_{(n_x*n_y*n_z)*(n_x*n_y*n_z)}[T_i]_{(n_x*n_y*n_z)*1} = [B]_{(n_x*n_y*n_z)*1}$$

where A and B are matrices having elements which are independent of the temperature of the insulation.

3. **Fluid**

The governing equation for a fluid in a particular duct is :

$$\frac{\dot{m}}{N}C_p\frac{dT_f}{dy} = h_f\frac{\pi D_i}{2}[(T_p - T_f) + (T_d - T_f)] \tag{4.2}$$

$$\frac{dT_f}{dy} = \frac{h_f \pi D_i N}{2\dot{m}C_p}[(T_p - T_f) + (T_d - T_f)] = U(y) \tag{4.3}$$

where,'U' is a function of 'y' for a particular duct.

Therefore,

$$T_f = \int U(y)dy + constant \tag{4.4}$$

For solving the above integral equation we have different methods available, the easiest of which is the Trapezoidal rule. By this method temperature of fluid at all nodal points inside the duct can easily be calculated.The temperature of the j^{th} node can be calculated by using the following equation :

$$T_f(j) = T_f(0) + \frac{\Delta y}{2}[U(1) + 2(U(2) + U(3) + ... + U(j-1)) + U(j)] \tag{4.5}$$

The outlet fluid temperature is calculated by taking the average of outlet temperature of fluid in different ducts.The efficiency (η) is defined as the ratio of total energy gained by the fluid to the total energy incident on the absorber plate.Thus,

$$\eta = \frac{\dot{m}C_p(T_{fo} - T_{fi})}{I_o A_p} \tag{4.6}$$

4. **Duct**

The heat equation(3.21) for a duct can be discretized in terms of temperatures at the nodal points.We obtain equations for each duct. Adiabatic boundary conditions are applied to the boundaries.Therefore, at boundaries we have,

$$\text{for} \quad y = 0 \quad T_d(-1) = T_d(1)$$

$$\text{for} \quad y = L \quad T_d(n_y + 1) = T_d(n_y - 1)$$

The equations obtained for each nodal point are related to each other.They can be written in a matrix form which is then solved by Gauss elimination method.Hence,

$$[A]_{(N*n_y)*(N*n_y)}[T_d]_{(N*n_y)*1} = [B]_{(N*n_y)*1}$$

5. **Plate**

The governing heat equation for the plate(3.22) is different for different zones depending upon whether it is in contact with the insulation or the fluid.

- For points in contact with the fluid

$$k_p[\frac{\partial^2 T_p}{\partial x^2} + \frac{\partial^2 T_p}{\partial y^2}] + \frac{q_{f-p}}{\delta_p} + \frac{q_{c-p}}{\delta_p} + \frac{S}{\delta_p} = 0 \tag{4.7}$$

- For points in contact with the insulation

$$k_p[\frac{\partial^2 T_p}{\partial x^2} + \frac{\partial^2 T_p}{\partial y^2}] + \frac{q_{i-p}}{\delta_p} + \frac{q_{c-p}}{\delta_p} + \frac{S}{\delta_p} = 0 \tag{4.8}$$

These equations can be discretized by using finite difference equations.Adiabatic boundary conditions have been assumed.Hence on boundaries we have

$$for \quad x = 0 \quad T_p(-1, j) = T_p(1, j)$$

$$for \quad y = 0 \quad T_p(i, -1) = T_p(i, 1)$$

$$\begin{array}{llll} for & x = W & & T_p(n_x+1,j) = T_p(n_x-1,j) \\ for & y = L & & T_p(i,n_y+1) = T_p(i,n_y-1) \end{array}$$

The matrix equation formed in this case is

$$[A]_{(n_x*n_y)*(n_x*n_y}[T_p]_{(n_x*n_y)*1} = [C]_{(n_x*n_y)*1} + [D]_{(1*1)}[T_p]^4_{(n_x*n_y)*1}$$

where A,C and D are matrices having elements which are independent of the temperature of the plate.

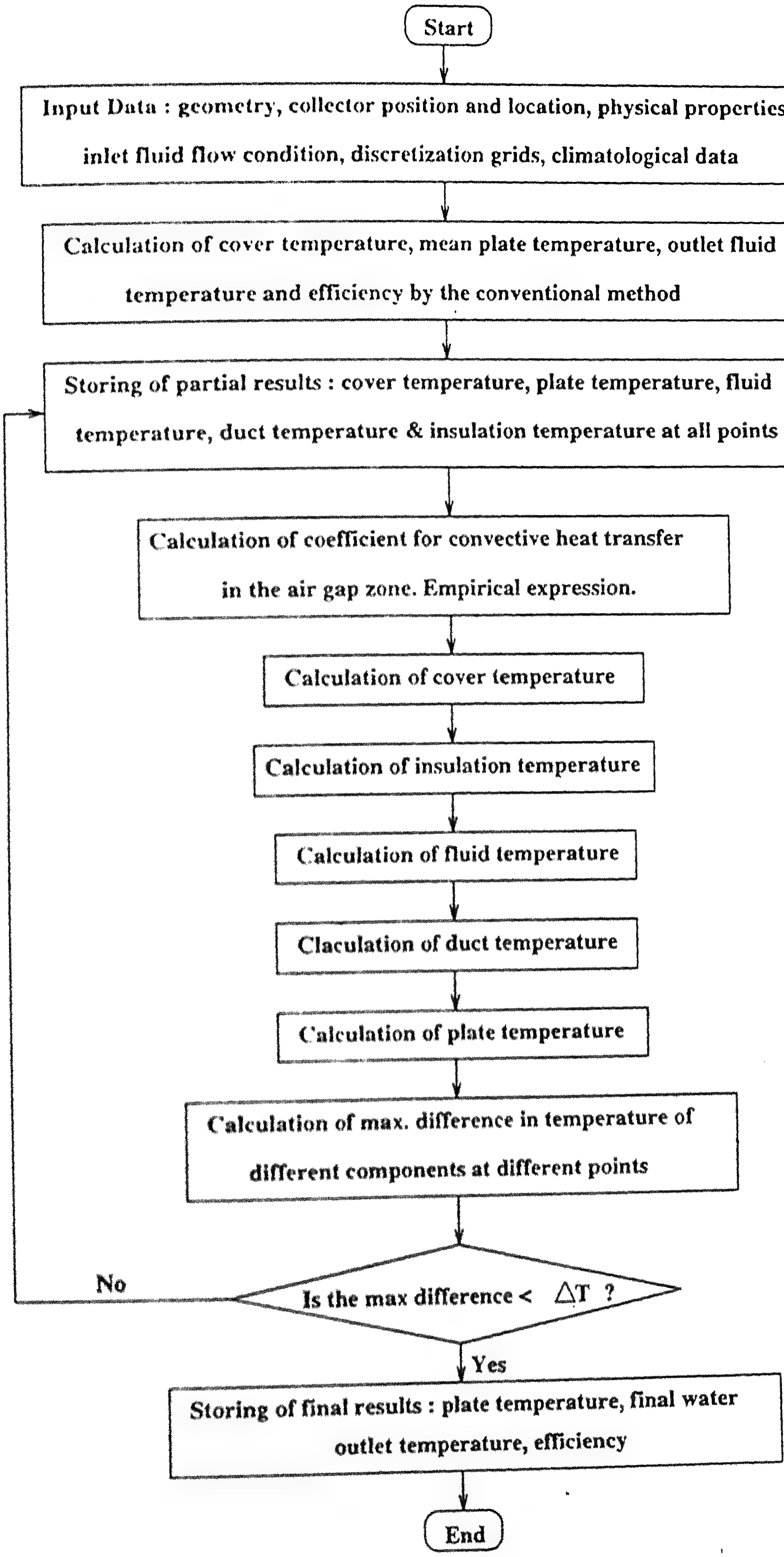

Figure 4.2: Computer programme flow chart

Chapter 5

Results And Discussion

To compute the outlet fluid temperature for a given geometry,position and location of a collector,there are three inputs, namely, inlet fluid temperature, mass flow rate and time of the day. The computation of the temperature of the various parts and fluid has been done for I^{st} of March, 1996. The flow in all the ducts has been combined in parallel in preference to the series connection. It is found, by the conventional method, that there are more losses in series connection which leads to a lower outlet fluid temperature as compared to parallel connection. This is because, as the temperature increases the losses increase too. The temperatures of the cover,the plate and the fluid have been calculated by using the computer program as discussed in Appendix A. The inlet fluid temperature is $21°C$ and the ambient temperature equal to $30°C$.

Figs. 5.1 and 5.2 give the variation of the rise in the fluid temperature with the time of the day for different mass flow rates. It is clear from the graphs that there is a wide difference between the results yielded by the multidimensional and the conventional approaches. For a mass flow rate of 18 kg/h the maximum temperature rise obtained is about $29°C$ by the multidimensional method while it is only $26°C$

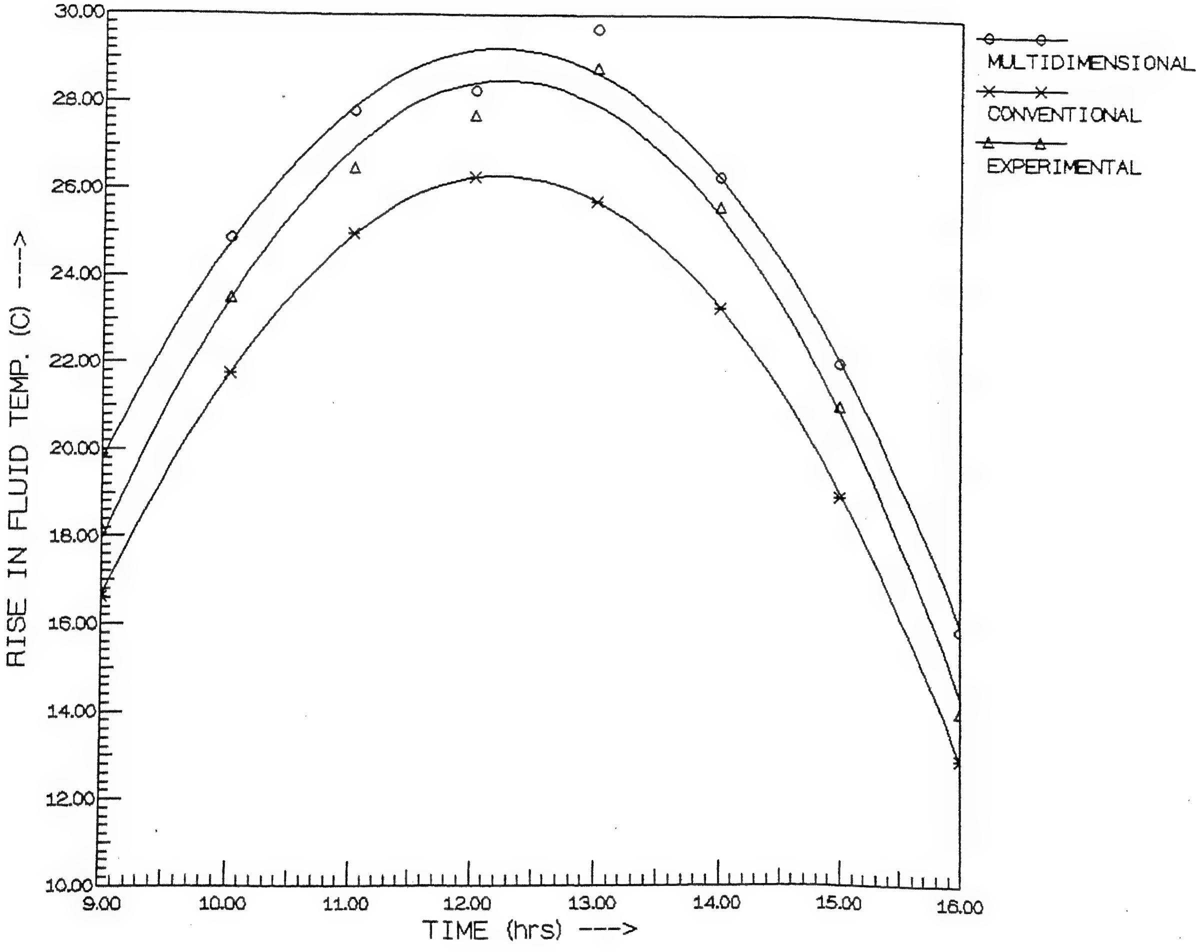

Figure 5.1: Rise in fluid temperature with the time of the day for a water flow rate of 18kg/h through the collector tubes

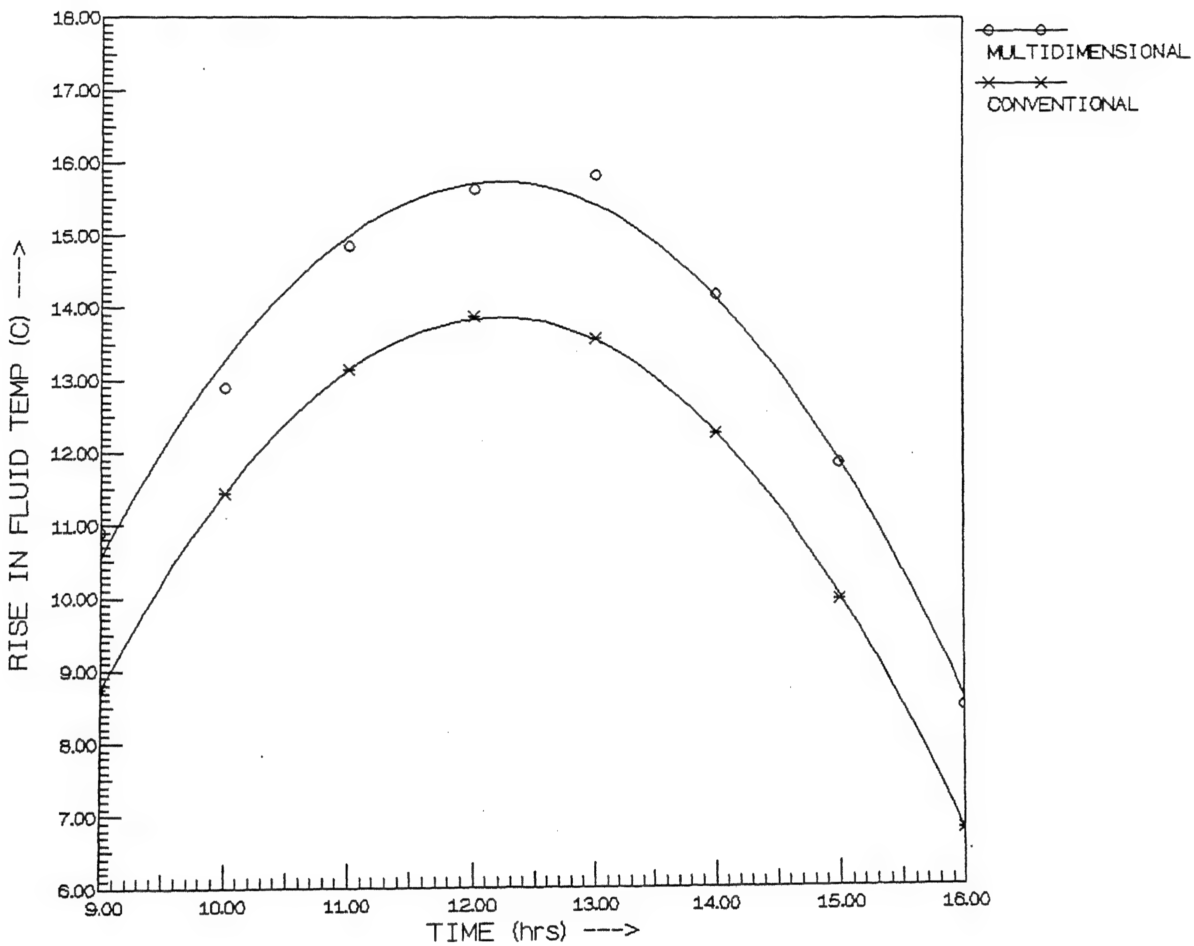

Figure 5.2: Rise in fluid temperature with the time of the day for a water flow rate of 36kg/h through the collector tubes

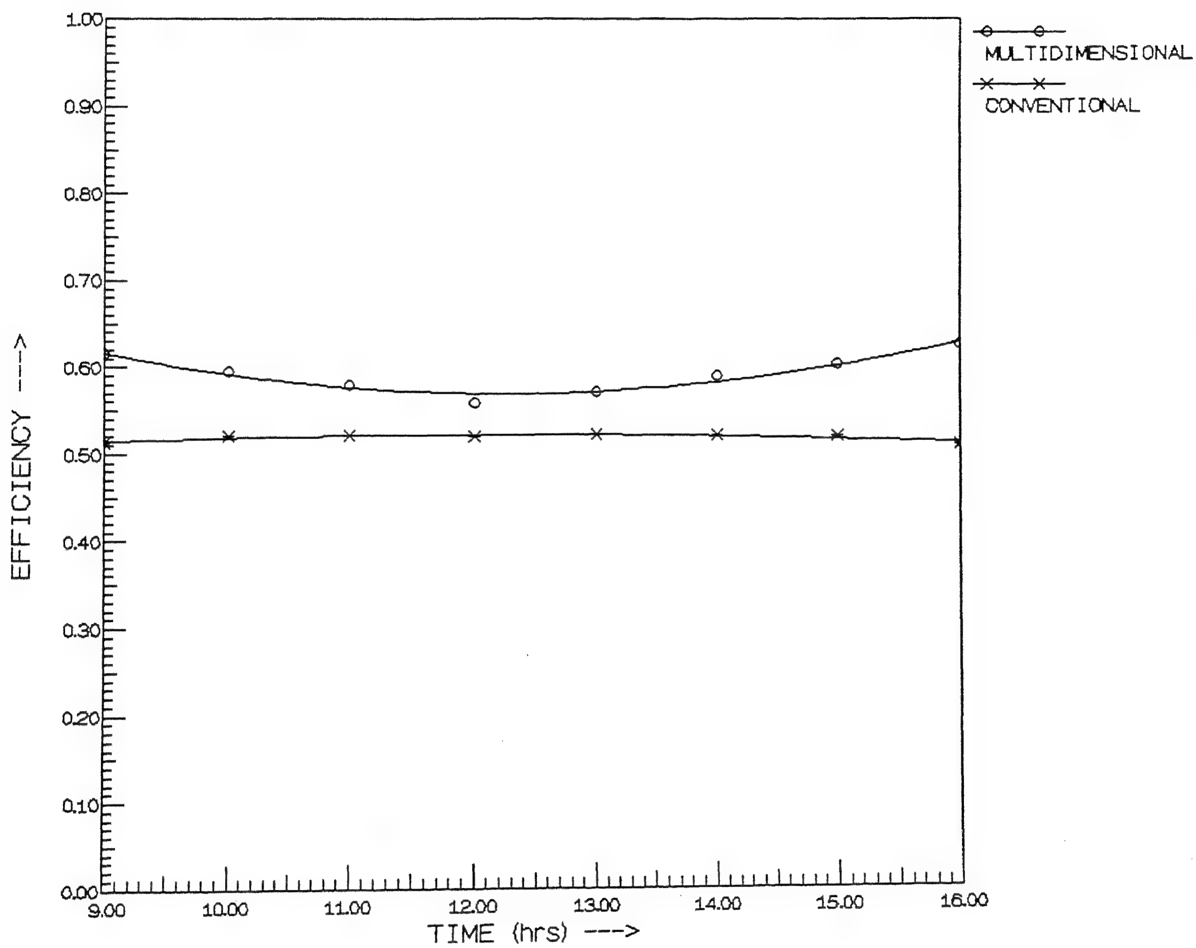

Figure 5.3: Variation of instantaneous efficiency with the time of the day for a water flow rate of 18 kg/h through the collector tubes

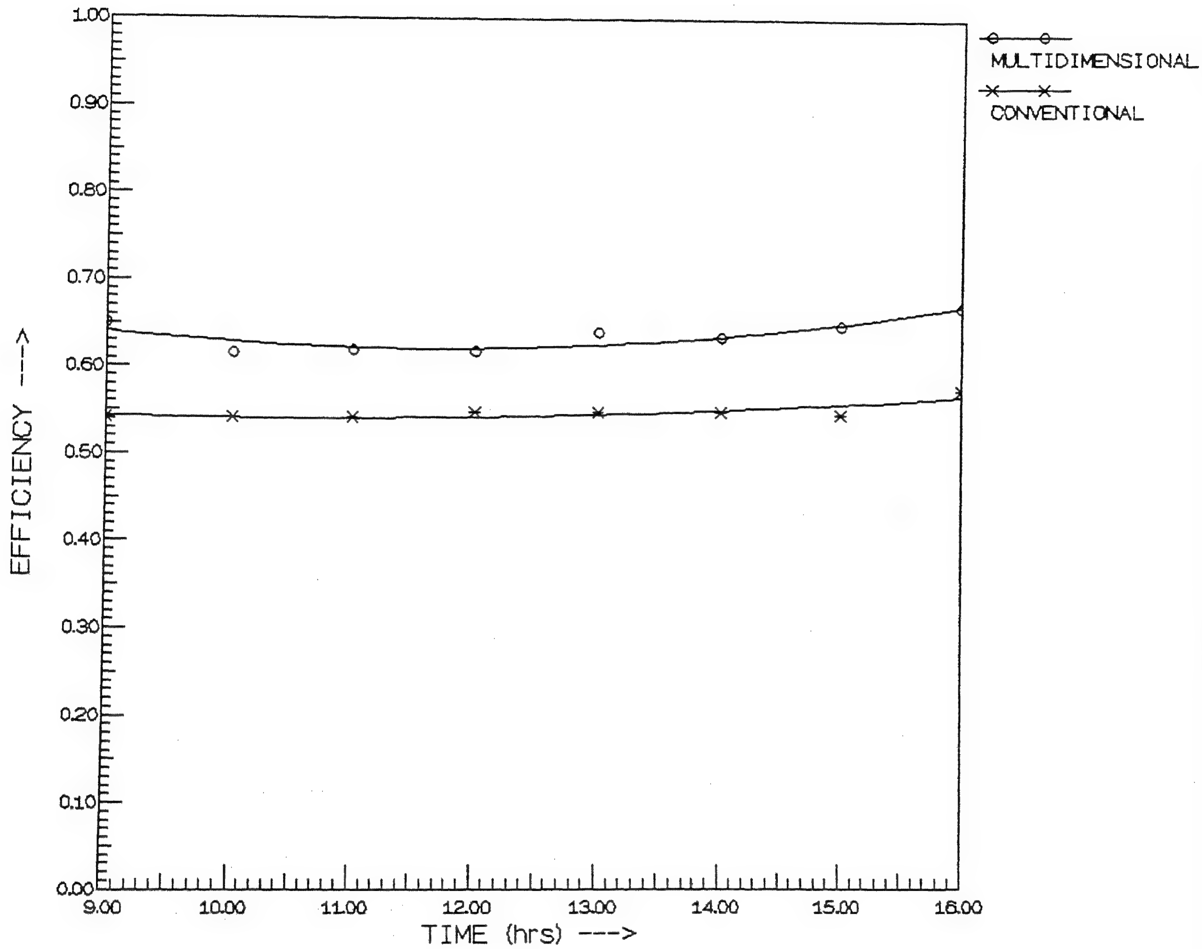

Figure 5.4: Variation of instantaneous efficiency with the time of the day for a water flow rate of 36 kg/h through the collector tubes

by the conventional method. For a mass flow rate of 36 kg/h the above values are 15.5^oC and 13.5^oC for multidimensional and conventional analyses respectively. Thus the multidimensional approach leads to a reduction of the conventional approach flat plate collector area by 8 to 10 percent for the same fluid temperature rise. Thus the collector becomes smaller and lighter. It is also clear from the graphs that the difference between the two approaches is almost the same for all times of the day i.e. independent of the amount of radiation incident on the plate. In Fig. 5.1, the experimental values taken from [17] have also been plotted. The figure shows that the experimental values are higher than those obtained by the conventional analysis. The experimental curve is, therefore, much closer to the curve yielded by the multidimensional approach. The discrepancy between the experimental values and the values from the multidimensional analysis could be due to various reasons ,viz., the insolation impinging on the plate may be less than the monthly average assumed in the present analysis, errors in measurement of fluid temperatures, variation of the property values with temperature etc.

Figs. 5.3 and 5.4 shows the variation of the efficiency with the time of the day for two different mass flow rates. The graphs show that the efficiency obtained by the multidimensional approach is more compared to that yielded by the conventional method. The difference in efficiency between the two analyses is about 7 to 10 percent which is quite significant. This difference in efficiency is expected as there is a significant difference in the rise of the fluid temperature for the two methods.It is also seen from the graphs that there is a dip in efficiency at about noon when radiation is maximum. This is because of the fact that as the radiation increases, the temperature increases and the losses increase too.

Figure 5.5 displays the rise in fluid temperature with mass flow rate for the two analyses. The rise is higher for the multidimensional analysis. The rise in fluid

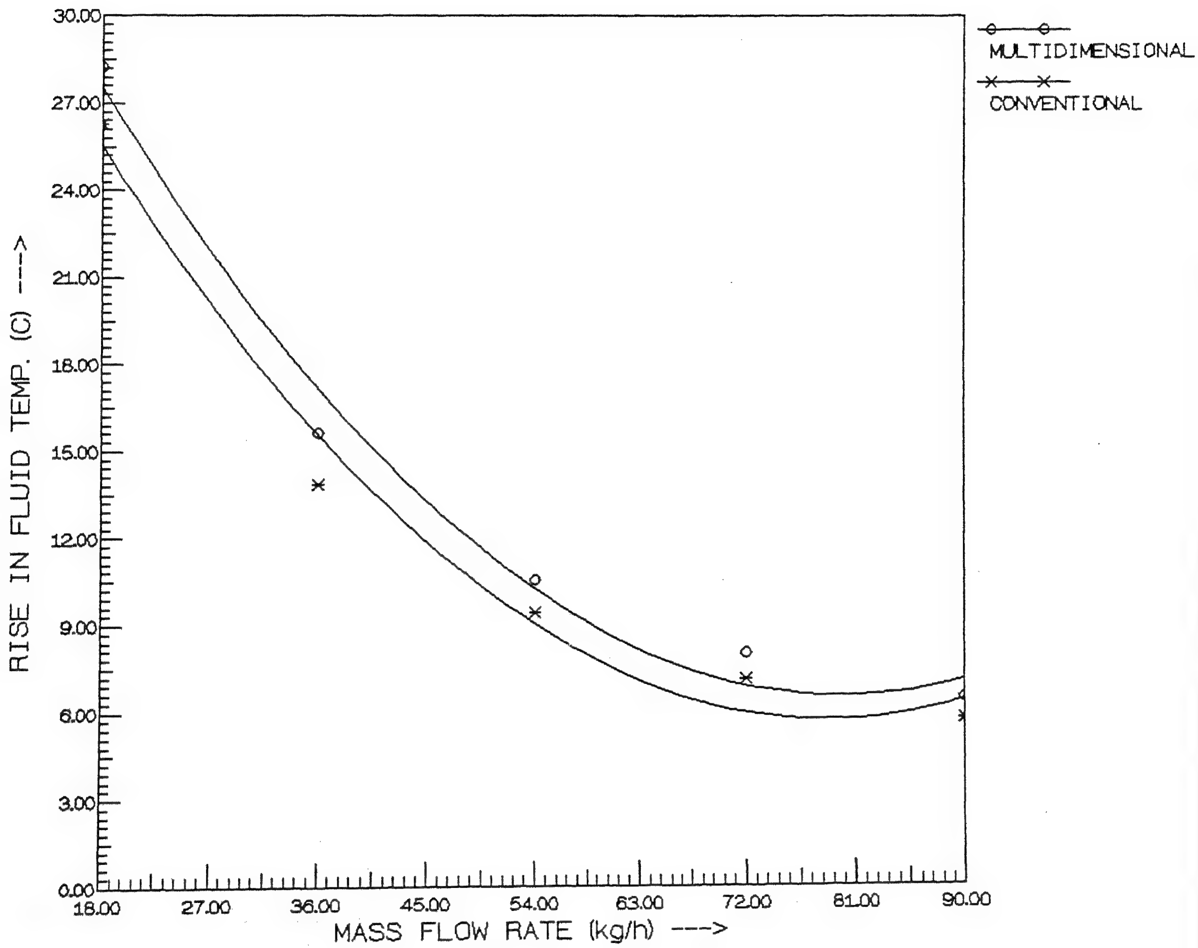

Figure 5.5: Rise in the fluid temperature with mass flow rate

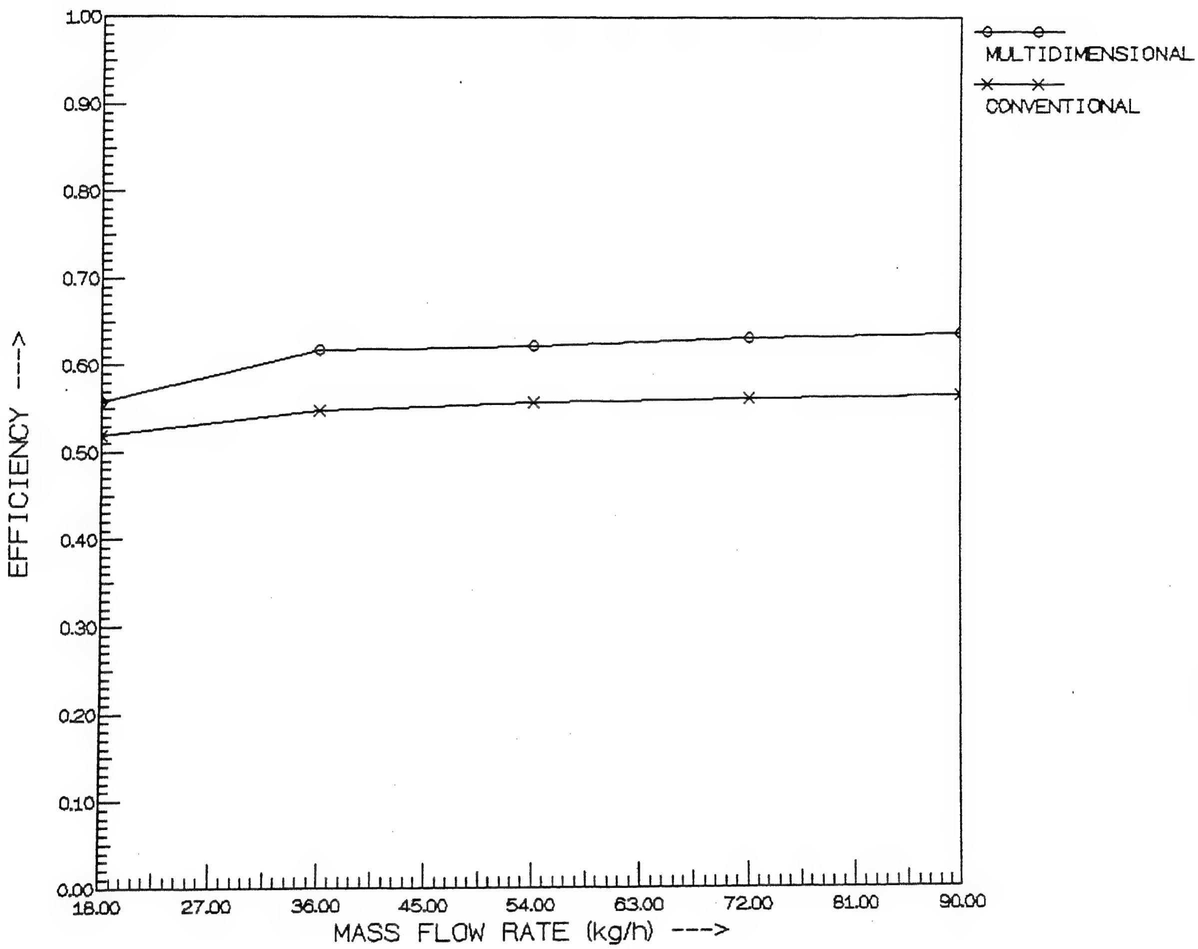

Figure 5.6: Variation of efficiency with mass flow rate of water

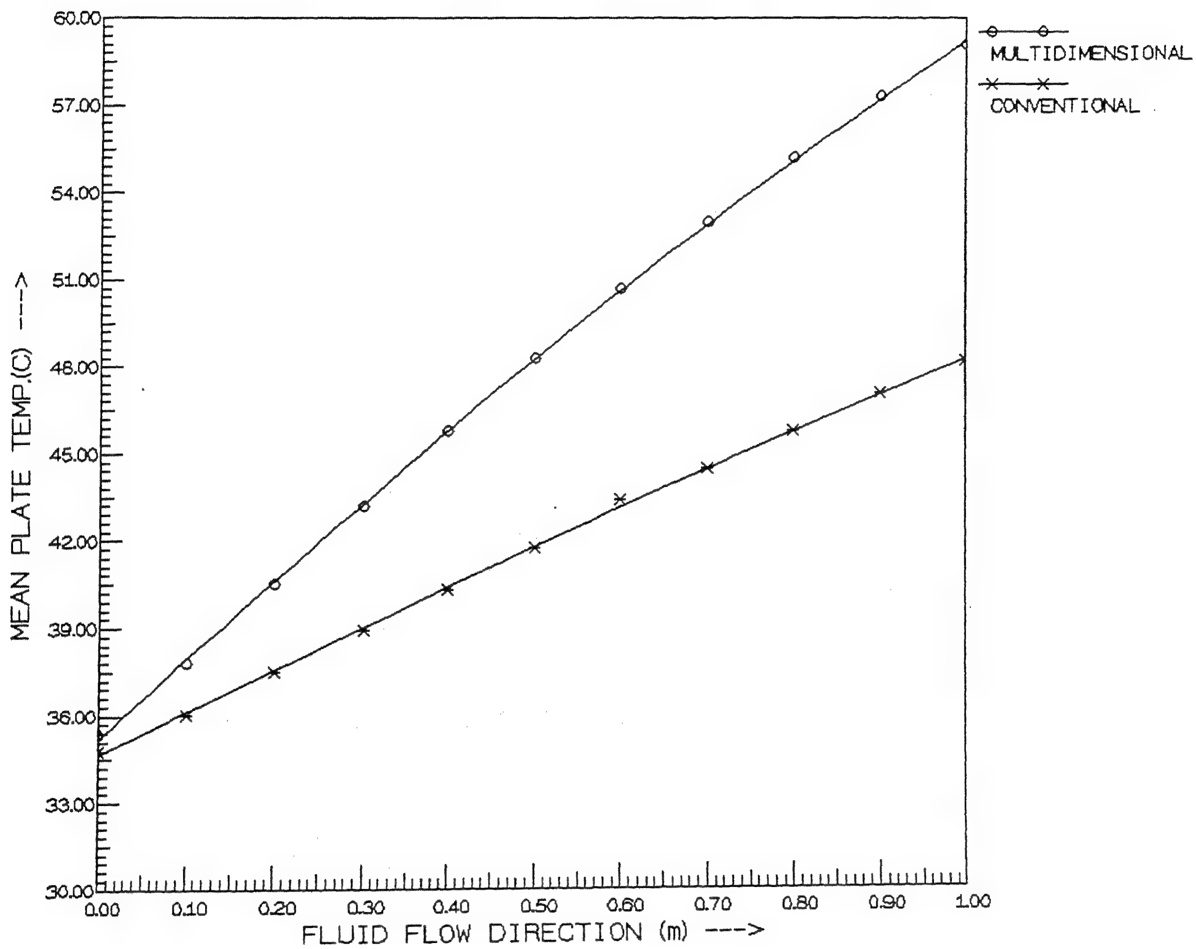

Figure 5.7: Variation of mean plate temperature along the fluid flow direction ($\dot{m} = 18$kg/h)

temperature continuously decreases as the mass flow rate is increased. This decrease is higher for the low mass flow rate region than for the higher mass flow rate region. This is because the amount of heat entering into the fluid is constant.

Figure 5.6 compares the variation of efficiency with mass flow rate for the multidimensional and the conventional methods. The curve for the multidimensional analysis shows that the efficiency increases to begin with and becomes almost constant towards larger values of mass flow rate while the curve for the conventional analysis does not show any variation with mass flow rate. This makes it clear that for the optimum utilisation of the solar flat plate collector, the mass flow rate should be greater than 40 kg/h approximately.

Figure 5.7 demonstrates the variation of plate temperature in the direction of the fluid flow for the two analysis. The plate temperature increases along the direction of the fluid flow because of the fact that at the start the temperature of fluid is low which increases as it flows through the flat plate collector. The graph also shows a remarkable difference in the values of plate temperature at different points along the direction of flow. Initially, the difference is small, but it gradually increases and becomes significantly high towards the end. The difference at the end of the flow is about $11^{\circ}C$.

Chapter 6

Conclusions

6.1 Conclusions

From the results of the present study, the following inferences may be drawn :

- The multidimensional approach for the analysis of a flat plate collector gives a higher outlet fluid temperature compared to the conventional approach, for the same inlet fluid temperature.

- The efficiency of the solar flat plate collector by the multidimensional approach is about 7 to 10 percent more than the conventional approach.

- For efficient utilisation of the flat plate collector the mass flow rate should be greater than 40 kg/h approximately.

- There is a larger plate temperature gradient in the direction of flow for the multidimensional approach in comparison to the conventional approach.

- Multidimensional approach leads to a reduction of the conventional approach flat plate collector area by 8 to 10 percent for the same fluid temperature rise. Thus the collector becomes smaller and lighter.

6.2 Suggestions for future work

The present work is a comparative study into the performance of a solar flat plate collector under multidimensional and conventional analyses. The next step which follows naturally from the present work would be to experimentally verify the results obtained in this study.

The variation in the values of physical properties with temperature of the materials involved has not been considered in this study. This could be taken up for investigation. Also the analysis presented herein assumes steady state conditions. A multidimensional, transient analysis of the collector with properties varying with temperature would be the next logical step for a complete theoretical analysis of a solar flat plate collector followed by its experimental verification.

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Appendix A

The following is the programme for the multi-dimensional solution of a solar flat plate collector. It includes different procedures for calculating the temperatures for different parts of the solar flat plate collector .

```
#include <stdio.h>
#include <math.h>
#include "ad.h"


#define nx 28
#define ny 11
#define nz 7
#define Length 1
#define Width 1
#define sigma    5.67e-8
#define epsilon  .01

double delta_p, delta_c,delta_d, Delta_x,Delta_y,Delta_z;
double Tc[nx][ny],Tp[nx][ny],Ti[nx][ny][nz],Tf[N][ny],Td[N][ny];
```

```
double Tp_old[nx][ny],Tf_old[N][ny],Td_old[N][ny],Tc_old[nx][ny],Ti_eq[nx][ny]
double T_amb,T_sky;
double k_p,k_c,k_d,k_i,Cp,Ep,Ec,h_f,h_w,Tpm,Tcm,Tdm,Q_abs,Io,S,hp_c,Tfo;
double Time,m,Tfi;


main()
{
    int i,j;
    double tmp;
    FILE *of;
    of = fopen("OUT","w");

    initiate1();
    printf("\nGive time of day in hours, mass flow rate,inlet fluid temp\n");
    scanf("%lf",&Time);
    scanf("%lf",&m);
    scanf("%lf",&Tfi);
    radiation();
    initiate2();
    do
    {
        coefficient();
        initiate3();
        cover();
        fprintf(of,"Tc[0][0] is %lf\n",Tc[0][0]);
        insulation();
```

```
        fprintf(of,"Ti[4][3][1] is %lf\n",Ti[4][3][1]);
        fluid();
        fprintf(of,"Tf[1][4] is %lf\n",Tf[1][4]);
        duct();
        fprintf(of,"Td[1][4] is %lf\n",Td[1][4]);
        plate();
        fprintf(of,"Tp[5][5] is %lf\n",Tp[5][5]);
        tmp = difference();
        fprintf(of,"difference is %lf\n",tmp);
    }
    while (tmp >  epsilon);
    printf("\n outlet temperature %lf\n",Tfo);
    fprintf(of,"\n  outlet temperature %lf\n",Tfo);
    return 0;
}
```

Appendix B

The values of the constants used, have been listed below :

Thickness of the glass cover(δ_c)	2 mm
Thickness of the absorber plate(δ_p)	2 mm
Thickness of the duct(δ_d)	2 mm
Thickness of the insulation(δ_i)	7 cm
Thermal conductivity of the glass cover(k_c)	0.76 $W.m^{-1}.K^{-1}$
Thermal conductivity of the copper absorber plate(k_p)	35 $W.m^{-1}.K^{-1}$
Thermal conductivity of the copper duct(k_d)	35 $W.m^{-1}.K^{-1}$
Thermal conductivity of the glass wool insulation(k_i)	0.03 $W.m^{-1}.K^{-1}$
Emissivity of the carbon black absorber plate(ϵ_p)	0.92
Emissivity of the glass cover(ϵ_c)	0.88
Heat transfer coefficient of the fluid(h_f)	205 $W.m^{-2}.K^{-1}$
Heat transfer coefficient of the wind(h_w)	8 $W.m^{-2}.K^{-1}$
Reflectivity of the surrounding (ρ)	0.2
Absorptivity of the carbon black absorber plate(α)	0.9
Collector tilt(β)	25^o
Latitude of Kanpur(ϕ)	26.28^o

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